TRANSIENT CREEP OF AN ICE SHEET LYING ON A HYDRAULIC BASE UNDER A CONCENTRATED LOAD

V. M. Aleksandrov and A. A. Shmatkova

UDC 624.131+539.215

An integrodifferential equation is derived for the bending of a beam (plate) under conditions of nonlinear transient creep. The problem of transient creep of a kink in an ice sheet lying on a hydraulic base and subjected to a concentrated load is investigated. The investigation was performed in two dimensions. The well-known model of Glen [1] is used to describe prolonged loading of ice. A numerical example is given.

1. We shall study transient creep in a beam undergoing bending under a prescribed constant load. The deflection is assumed to be downward. The x axis is oriented along the axis of the beam and the z axis is oriented vertically downward. The relative elongation e of a fiber in the beam consists of the elastic elongation e_1 and the creep strain e_2:

$$e = e_1 + e_2, \tag{1.1}$$

and the elastic elongation $e_1 = \sigma(t)/E$ [$\sigma(t)$ is the normal stress and E is the elastic modulus]. We shall assume that the creep strain is a power-law function of $\sigma(t)$, namely,

$$e_2 = \int\limits_0^t B(t-\tau)\,\sigma^m(\tau)\,d\tau.$$

Here m is the creep index, a dimensionless constant; for ice $m > 1$. The function $B(t - \tau)$ is the creep kernel, having the dimensions $(\mathrm{kg/m^2})^{-m}\cdot\mathrm{sec}^{-1}$. It is a positive decreasing function of time, measured from the onset of creep, and asymptotically approaches the limiting value $B_\infty = \mathrm{const}$. Using for $B(t - \tau)$ the experimentally obtained curve for ice [1], we assume that

$$B(t-\tau) = B_\infty + B_0 \exp\left[-\mu(t-\tau)\right]. \tag{1.2}$$

Since the sign of e and therefore the sign of σ will depend on the sign of z, we represent Eq. (1.1), based on everything we have said so far, in the form

$$e = \sigma(t)/E + \int\limits_0^t B(t-\tau)\,|\sigma(\tau)|^{m-1}\sigma(\tau)\,d\tau. \tag{1.3}$$

The relative elongation of a fiber in the beam at a distance z from the neutral axis, according to the hypothesis of plane sections, $e = z/\rho$ (ρ is the radius of curvature of the neutral layer). When the deflection is small it can be assumed, with adequate accuracy, that $e = z d^2v/dx^2$. Making this substitution in Eq. (1.3) we obtain

$$z\frac{d^2v}{dx^2} = \frac{\sigma}{E} + \int\limits_0^t B(t-\tau)\,|\sigma|^{m-1}\sigma\,d\tau. \tag{1.4}$$

Let Ω be the area of the transverse cross section of the beam. Then in some section of the beam the bending moment $M = \int\limits_\Omega z\sigma\,d\Omega$. Multiplying Eq. (1.4) by $zd\Omega$ and integrating over the transverse cross section we find

$$I\frac{d^2v}{\partial x^2} = \frac{M}{E} + \int\limits_0^t B(t-\tau)\int\limits_\Omega |\sigma|^{m-1}\sigma z\,d\Omega\,d\tau, \tag{1.5}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 132-138, September-October, 1990. Original article submitted October 31, 1988; revision submitted March 28, 1989.

where $I=\int\limits_{\Omega} z^2 d\Omega$ is the moment of inertia of the transverse cross section. Since σ and z always have the same sign, the inner integral in the relation (1.5) can be written as

$$\int\limits_{\Omega} |\sigma|^m |z|\, d\Omega. \tag{1.6}$$

Let $m^{-1} + n^{-1} = 1$. Then, by virtue of Hölder's inequality,

$$\int\limits_{\Omega} |z||\sigma|\, d\Omega \leqslant \left[\int\limits_{\Omega} (|z|^{1/m}|\sigma|)^m d\Omega\right]^{1/m} \left(\int\limits_{\Omega} |z|\, d\Omega\right)^{1/n} = C_1 \left(\int\limits_{\Omega} |z||\sigma|^m d\Omega\right)^{1/m},$$

$$C_1^{-m} M^m \leqslant \int\limits_{\Omega} |\sigma|^m |z|\, d\Omega.$$

Thus, the lower bound on the integral (1.6) has been obtained. To obtain the upper bound, we use Favard's inequality [2]

$$\frac{1}{\omega}\int\limits_{\omega} f^p d\omega \leqslant \frac{2p}{p+1}\left(\frac{1}{\omega}\int\limits_{\omega} f\, d\omega\right)^p$$

($p > 1$; f is a nonnegative continuous concave function, not equal identically to zero on ω). We note that

$$\int\limits_{\Omega} |\sigma|^m |z|\, d\Omega = \int\limits_{\Omega_1} |\sigma|^m d\Omega_1,$$

where $m > 1$ and $|\sigma|$ is a concave function. Then

$$\int\limits_{\Omega_1} |\sigma|^m d\Omega_1 \leqslant \frac{2m}{(m+1)\,\Omega_1^{m-1}}\left(\int\limits_{\Omega_1} |\sigma|\, d\Omega_1\right)^m,$$

or

$$\int\limits_{\Omega} |\sigma|^m |z|\, d\Omega \leqslant \frac{2m}{(m+1)\,\Omega_1^{m-1}}\left(\int\limits_{\Omega} |\sigma||z|\, d\Omega\right)^m = C_2^{-m} M^m.$$

Therefore,

$$C_1^{-m} M^m \leqslant \int\limits_{\Omega} |\sigma|^m |z|\, d\Omega \leqslant C_2^{-m} M^m.$$

We shall calculate the constants C_1 and C_2 for a beam with a rectangular profile of unit width and height h:

$$C_1^{-m} = (2/h)^{2(m-1)}, \quad C_2^{-m} = 2mC_1^{-m}/(m+1).$$

We introduce

$$C^* = (C_1^{-m} + C_2^{-m})/2 = (1+3m)\, C_1^{-m}/2\,(m+1),$$

Then it is easy to show that for m ranging from 1 to 13 the ratio $C^*/C_1^{-m} = (1\text{-}1.43)$ and $C^*/C_2^{-m} = (1\text{-}0.77)$. So, it can be assumed approximately that

$$\int\limits_{\Omega} |\sigma|^m |z|\, d\Omega \simeq C^* M^m. \tag{1.7}$$

Using Eq. (1.7) we transform the relation (1.5) into the form

$$I\frac{d^2 v}{dx^2} = \frac{M}{E} + C^* \int\limits_0^t B(t-\tau)\, M^m d\tau. \tag{1.8}$$

Let q(x) be the intensity of the distributed load and Q(x) the shearing force in some section of the beam. Then q(x), Q(x), and M(x), according to the theory of Zhuravskii–Shvedler, are related by known relations. Based on this we write

$$d^2M/dx^2 = -q,\ dM/dx = Q. \tag{1.9}$$

We note that q(x) is the per unit length pressure transferred from the beam to the base. Let q(x) at any point be proportional to the deflection of the beam. Denoting by k the coefficient of proportionality (Winkler's bed constant of the base), we rewrite the first formula in Eq. (1.9) as

$$d^2M/dx^2 = -kv. \tag{1.10}$$

Differentiating this relation twice we obtain

$$d^2v/dx^2 = (-1/k)d^4M/dx^4. \tag{1.11}$$

Substituting Eq. (1.11), the relation (1.8) assumes the form

$$\frac{d^4M}{dx^4} + \frac{k}{EI}M + \frac{kC^*}{I}\int_0^t B(t-\tau)\,M^m(x,\tau)\,d\tau = 0. \tag{1.12}$$

Thus, we have derived an approximate integrodifferential equation for the bending moment in the case of transient creep in a beam lying on a uniform base. It should be noted that Eq. (1.12) also holds for an infinite plate subjected to cylindrically bending; it is merely necessary to replace EI by the flexural rigidity D.

2. We shall examine the case of the bending of a beam of thickness lying on a hydraulic base, which we shall model by a Winkler base. Let the density of the liquid ρ^* = const. The $k = \rho^* g$, where g is the acceleration of gravity. The bending is caused by a concentrated force G. We assume that the law (1.3) can be used to describe the material of the beam. Then the bending problem reduces to the integrodifferential equation (1.12). We write the boundary conditions of the problem: The deflection and its first derivative vanish at infinity

$$v(\pm\infty) = 0; \tag{2.1}$$

$$v'(\pm\infty) = 0. \tag{2.2}$$

The curved axis of the beam must have a horizontal tangent at the point x = 0:

$$(v')_{x=0} = 0. \tag{2.3}$$

The transverse force for the right side of the beam at x = 0 is

$$(M')_{x=0} = -G/2. \tag{2.4}$$

The initial condition is that at t = 0 the classical instantaneous elastic solution is obtained. As a result of the symmetry it is sufficient to study only the part of the beam to the right of the applied force. We introduce the reduced time

$$\zeta = 1 - e^{-\mu t},\ \zeta \in [0, 1),\ \vartheta = 1 - e^{-\mu\tau},\ \vartheta \in [0, 1), \tag{2.5}$$

where the parameter μ is the same parameter as in the relation (1.2), and is determined experimentally. With the help of Eq. (1.2) and (2.5) we transform Eq. (1.12) into the form

$$\frac{d^4M}{dx^4} + \frac{k}{EI}M + \frac{kC^*}{I\mu}\left[B_\infty\int_0^\zeta \frac{M^m}{1-\vartheta}\,d\vartheta + B_0(1-\zeta)\int_0^\zeta \frac{M^m}{(1-\vartheta)^2}\,d\vartheta\right] = 0. \tag{2.6}$$

We shall seek the solution of Eq. (2.6) in the form of the series

$$M(x,\zeta) = \sum_{i=0}^{\infty} M_i(x)\,\zeta^i. \tag{2.7}$$

We shall study six terms in the expansion (2.7). Substituting Eq. (2.7) into Eq. (2.6), performing a series of transformations, and equating the expressions with identical powers of ζ^i, we arrive at the following system of equations:

$$M_0^{\mathrm{IV}} + J_1 M_0 = 0; \tag{2.8}$$

$$M_i^{\mathrm{IV}} + J_1 M_i = -(J_2 F_i + J_3 T_i),\quad i = 1, \ldots, 5. \tag{2.9}$$

Here we have introduced the following notation:

$$J_1 = k/EI,\quad J_2 = kC^*B_\infty/I\mu,\ J_3 = kC^*B_0/I\mu,$$
$$F_1 = f_0,\qquad T_1 = f_0,$$
$$F_2 = (1/2)(f_0 + f_1),\qquad T_2 = (1/2)f_1,$$
$$F_3 = (1/3)(f_0 + f_1 + f_2),\qquad T_3 = (1/6)(f_1 + 2f_2),$$
$$F_4 = (1/4)(f_0 + f_1 + f_2 + f_3),\qquad T_4 = (1/12)(f_1 + 2f_2 + 3f_3),$$
$$F_5 = (1/5)(f_0 + f_1 + f_2 + f_3 + f_4),\ T_5 = (1/20)(f_1 + 2f_2 + 3f_3 + 4f_4),$$

and

$$f_0 = M_0^m,\quad f_1 = M_0^m m(M_1/M_0),$$
$$f_2 = M_0^m [m(M_2/M_0) + (1/2)m(m-1)(M_1/M_0)^2],$$
$$f_3 = M_0^m [m(M_3/M_0) + m(m-1)(M_1/M_0)(M_2/M_0) +$$
$$+ (1/6)m(m-1)(m-2)(M_1/M_0)^3],$$
$$f_4 = M_0^m [m(M_4/M_0) + (1/2)m(m-1)(M_2/M_0)^2 + (1/2)m(m-1)(m-2)\times$$
$$\times (M_1/M_0)^2(M_2/M_0) + (1/24)m(m-1)(m-2)(m-3)(M_1/M_0)^4].$$

a) Zeroth Approximation. At t = 0 we obtain the classical instantaneous elastic solution

$$M_0 = (G/4\beta)e^{-\beta x}(\cos\beta x - \sin\beta x),$$
$$v_0 = (-G\beta/2k)e^{-\beta x}(\cos\beta x + \sin\beta x),\quad v_0' = (G\beta^2/k)e^{-\beta x}\sin\beta x. \tag{2.10}$$

b) First Approximation. We take two terms of the expansion (2.7). Then Eqs. (2.8) and (2.9) with i = 1 must be studied. Using the notation introduced and the results of the zeroth approximation, Eq. (2.9) with i = 1 can be written in the form

$$M_1^{\mathrm{IV}} + J_1M_1 = J_4e^{-\beta mx}\cos^m(\pi/4 + \beta x)$$
$$\left(J_4 = (-kC^*/I\mu)\left(G/2^{3/2}\beta\right)^m(B_0 + B_\infty)\right). \tag{2.11}$$

The solution of the homogeneous equation (2.11) is

$$M_1^* = e^{\beta x}(A_1\cos\beta x + B_1\sin\beta x) + e^{-\beta x}(C_1\cos\beta x + D_1\sin\beta x). \tag{2.12}$$

We find a particular solution of the inhomogeneous equation (2.11) by the method of variation of constants. The derivatives of the functions sought are determined from the corresponding system of equations, which are too complicated to write out here. To obtain the functions A_1, B_1, C_1, and D_1 themselves it is necessary to study four types of integrals:

$$\int\exp[-\beta(m+1)x]\cos\beta x\cos^m(\pi/4 + \beta x)dx =$$
$$= -[\sqrt{2}\beta(m+1)]^{-1}\exp[-\beta(m+1)x]\cos^{m+1}(\pi/4 + \beta x),$$
$$\int\exp[-\beta(m-1)x]\cos\beta x\cos^m(\pi/4 + \beta x)dx =$$
$$= -(\sqrt{2}\beta m)^{-1}\exp[-\beta(m-1)x]\cos^{m+1}(\pi/4 + \beta x) -$$
$$-m^{-1}\int\exp[-\beta(m-1)x]\sin\beta x\cos^m(\pi/4 + \beta x)dx,$$
$$\int\exp[-\beta(m+1)x]\sin\beta x\cos^m(\pi/4 + \beta x)dx =$$
$$= -(\sqrt{2}\beta)^{-1}\exp[(m+1)\pi/4][(m+1)^{-1}\exp[-(m+1)y]\cos^{m+1}y +$$
$$+ 2\int(\exp(-y)\cos y)^{m+1}dy],$$
$$\int\exp[-\beta(m-1)x]\sin\beta x\cos^m(\pi/4 + \beta x)dx =$$
$$= -(\sqrt{2}\beta)^{-1}\exp[(m-1)\pi/4][(m+1)^{-1}\exp[-(m-1)y]\cos^{m+1}y +$$
$$+2m(m+1)^{-1}\int\exp[-(m-1)y]\cos^{m+1}y\,dy].$$

Here we introduce the notation $y = \pi/4 + \beta x$. Using the recurrence relation from [3], we obtain

$$\int\exp[-(m-1)y]\cos^{m+1}y\,dy = [2(m^2+1)]^{-1}\{\exp[-(m-1)y]\cos^m y\times$$
$$\times[-(m-1)\cos y + (m+1)\sin y] + m(m+1)\int[\exp(-y)\cos y]^{m-1}dy\}.$$

Thus, all integrals reduce to the calculation of one main integral of the form $\int[\exp(-y) \times \cos y]^{\gamma}dy$, where $\gamma = \{(m + 1), (m - 1); m > 1\}$. Based on everything we have said so far, the general solution of Eq. (2.11) can be written as

$$M_1 = M_1^* + J_5\{e^{-my}\cos^{m+2}y + (m+1)e^{y}\sin y\int(e^{-y}\cos y)^{m+1}dy + \\ + e^{-y}(m\cos y + \sin y)\int e^{-(m-1)y}\cos^{m+1}y\,dy\}\quad (J_5 = J_4\exp(\pi m/4)/[4\beta^4(m+1)]^{-1}). \tag{2.13}$$

Using the relation (1.10), we find

$$v_1 = -k^{-1}d^2M_1/dx^2 = -2\beta^2k^{-1}[\exp(\beta x)(B_1\cos\beta x - \\ -A_1\sin\beta x) + \exp(-\beta x)(C_1\sin\beta x - D_1\cos\beta x)] - 2J_5\beta^2k^{-1}\times \\ \times[\exp(-my)\cos^{m+2}y + (m+1)\exp(y)\cos y\int[\exp(-y)\cos y]^{m+1}dy + \\ + \exp(-y)(m\sin y - \cos y)\int\exp[-(m-1)y]\cos^{m+1}ydy]. \tag{2.14}$$

It is not difficult to write a relation for the derivative of v_1:

$$v_1' = -2\beta^3k^{-1}\{\exp(\beta x)[(B_1 - A_1)\cos\beta x - (A_1 + B_1)\sin\beta x] + \\ + \exp(-\beta x)[(D_1 + C_1)\cos\beta x - (C_1 - D_1)\sin\beta x]\} - 2J_5\beta^3k^{-1}\times \\ \times\{-2\exp(-my)\cos^{m+1}y\sin y + (m+1)\exp(y)(\cos y - \sin y)\times \\ \times\int[\exp(-y)\cos y]^{m+1}dy + \exp(-y)[(m+1)\cos y - (m-1)\sin y]\times \\ \times\int\exp[-(m-1)y]\cos^{m+1}y\,dy\}. \tag{2.15}$$

Using the boundary conditions (2.1) and (2.2), we obtain

$$A_1 = B_1 = 0.$$

The condition (2.3) makes it possible to determine the constant C_1:

$$C_1 = -D_1 + \sqrt{2}J_5\{(\sqrt{2})^{-(m+1)}\exp(-\pi m/4) - \exp(-\pi/4)\times \\ \times[\int\exp[-(m-1)y]\cos^{m+1}ydy]_{y=\pi/4}\}. \tag{2.16}$$

With the help of Eq. (2.4) we find the constant D_1:

$$D_1 = -G/(4\beta) + J_5(\sqrt{2})^{-1}\{(\sqrt{2})^{-(m+1)}\exp(-\pi m/4) - \\ -[(m+1)\exp(\pi/4)\int[\exp(-y)\cos y]^{m+1}dy - (m-1)\exp(-\pi/4)\times \\ \times\int\exp[-(m-1)y]\cos^{m+1}ydy]_{y=\pi/4}\}. \tag{2.17}$$

Thus, we have obtained analytically the first approximation of the problem

$$M = M_0 + M_1\zeta,\ v = v_0 + v_1\zeta,\ v' = v_0' + v_1'\zeta,$$

where M_0, v_0, and v_0' are determined by the relations (2.10), M_1 is determined by the relations (2.12) and (2.13), v_1 is determined by Eq. (2.14), and v_1' is determined by Eq. (2.15); ζ is related with t by the dependence (2.5); and, the constants C_1 and D_1 are given by the formulas (2.16) and (2.17), respectively.

c) Higher-Order Approximations. The second and higher order (including fifth) approximations of the problem were obtained numerically. The right sides of Eqs. (2.9) with i = 2, ..., 5 were written out in detail. The system of the main equations and the boundary conditions were transformed into a form convenient for programming. The standard system arising when the boundary—value problems for fourth-order ordinary differential equations are approximated was obtained. The system was solved by Gauss' elimination method, based on which an algorithm, called the monotonic sweep algorithm [4], was written. A FORTRAN computer program for calculating the values of M_i and v_i of interest to us was written.

d) Limiting Solutions. We shall study the behavior of the solution of Eq. (1.12) for the bending moment in the limit $t \to \infty$.

1) Let in Eq. (1.2) $B_\infty = 0$, but $B_0 \neq 0$. Then, it can be assumed that

$$M(x,\ t) \to M_\infty(x),\ t \to \infty. \tag{2.18}$$

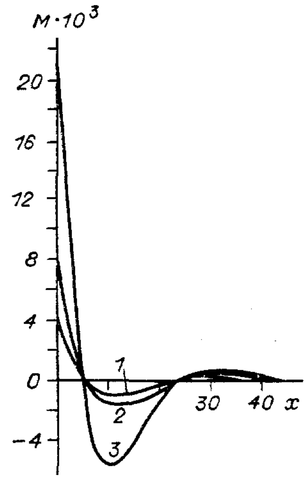

Fig. 1

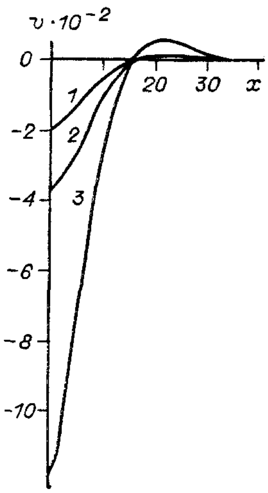

Fig. 2

Using Eq. (2.18), we obtain from Eq. (1.12)

$$M_\infty^{\mathrm{IV}} + (k/EI)\,M_\infty + (kC^*B_0/I)\,M_\infty^m \int_0^t e^{-\mu(t-\tau)}d\tau = 0.$$

Calculating the integral and passing to the limit $t \to \infty$, we find that the function $M_\infty(x)$ satisfies the equation

$$M_\infty^{\mathrm{IV}} + (k/EI)\,M_\infty + (kC^*B_0/I\mu)\,M_\infty^m = 0. \tag{2.19}$$

2) Assume that in Eq. (1.2) the constant $B_0 = 0$, but $B_\infty \neq 0$. We differentiate Eq. (1.12) once with respect to t:

$$\dot{M}^{\mathrm{IV}} + (k/EI)\dot{M} + (kC^*B_\infty/I)M^m = 0. \tag{2.20}$$

We shall seek the solution of this equation in the limit $t \to \infty$ in the form

$$M(x,t) \sim M_\infty(x)\,t^{-(m-1)^{-1}}, \quad m > 1. \tag{2.21}$$

Substituting Eq. (2.21) into Eq. (2.20), we find that in this case the function $M_\infty(x)$ satisfies the equation

$$M_\infty^{\mathrm{IV}} + (k/EI)\,M_\infty - (kC^*B_\infty/I)(m-1)\,M_\infty^m = 0. \tag{2.22}$$

To solve Eqs. (2.19) and (2.22) they must be supplemented by the relation (1.10) and the boundary conditions (2.1)-(2.4), respectively. Then the method of special orthonormal polynomials can be applied; the idea of these polynomials is briefly presented in [5].

3) Assume that in Eq. (1.2) both constants B_0 and B_∞ are different from zero. We introduce the notation $A = k/EI$, $B = kC^*B_\infty/I$, and $C = kC^*B_0/I$, and rewrite Eq. (1.12) in the form

$$M^{\mathrm{IV}} + AM + \int_0^t \left[B + Ce^{-\mu(t-\tau)}\right] M^m d\tau = 0. \tag{2.23}$$

Next, differentiating Eq. (2.23) once and twice with respect to t and eliminating from the relations obtained and Eq. (2.23) the integrals we arrive at the following differential equation:

$$\ddot{M}^{\mathrm{IV}} + A\ddot{M} + (B+C)mM^{m-1}\dot{M} + \mu(\dot{M}^{\mathrm{IV}} + A\dot{M} + BM^m) = 0. \tag{2.24}$$

Now, if we assume that as $t \to \infty$, M(x, t) satisfies the relation (2.18), then from Eq. (2.24) we find $M_\infty(x) \equiv 0$. Therefore, it must be assumed that as $t \to \infty$ the function M(x, t), which is a solution of Eq. (2.24), approaches zero. The solution of Eq. (2.24) for large t can be sought in the form, for example, of a series

$$M(x,t) = \sum_{i=1}^{\infty} M_i(x)\,e^{-i\nu t}, \tag{2.25}$$

if m is an integer (ν is an arbitrary constant). Substituting Eq. (2.25) into Eq. (2.24) and equating the terms with like powers of $\exp(-i\nu t)$, we obtain an infinite system of linear differential equations which must be solved successively to determine the functions $M_i(x)$.

e) Example. To illustrate the problem examined above we shall present a numerical example. We take the experimental values from [1]. The following starting data are provided to the program: $t = (0\text{-}15)$ sec; $E = 4\cdot 10^9$ kg/m·sec^2; $G = 2500$ kg; ($B_0 = 84\cdot 10^{-8}$, $B_\infty = 5.6\cdot 10^{-8}$) $(\text{kg/m}^2\cdot\text{sec}^2)^{-m}\text{sec}^{-1}$; $\mu = 3\cdot 10^{-2}$ sec^{-1}; $m = 1.72$; $h = 0.25$ m; $\rho^* = 10^3$ kg/m^3; $g = 9.81$ m/sec^2.

Figures 1 and 2 show the curves of the approximation for the bending moment M and the approximations for the deflection v as a function of the coordinate x for a fixed time t = 15 sec. The curves 1 correspond to the zeroth approximation of the problem (classical instantaneous elastic solution) and the curves 2 are the first approximation of the problem. These curves were calculated using the formulas obtained analytically (the points a and b above). It was established numerically that for the time t = 15 sec five approximations of the problem are adequate (curve 3). As the time increases more terms must be retained in the series expansion (2.7).

LITERATURE CITED

1. D. E. Hasin, Dynamics of Ice Sheets [Russian translation], Gidrometeoizdat, Leningrad (1967).
2. E. F. Beckenbach and R. Bellman, Inequalities, Springer-Verlag, New York (1965).
3. I. S. Gradshtein and I. M. Ryzhik, Tables of Integrals, Sums, Series, and Products [in Russian], Nauka, Moscow (1971).
4. A. A. Samarskii and E. S. Nikolaev, Methods for Solving Finite-Difference Equations [in Russian], Nauka, Moscow (1978).
5. V. M. Aleksandrov, N. Kh. Arutyunyan, and A. A. Shmatkova, "Creep and strength of ice sheets under loads and stamps," in: Electrophysical and Physicomechanical Properties of Ice [in Russian], Gidrometeoizdat, Leningrad (1989).

FLOW AND SEPARATION OF A RAREFIED BINARY GAS MIXTURE IN A CYLINDRICAL GAP WITH SUPERSONIC ROTATION OF THE OUTER CYLINDER

V. D. Borisevich and S. V. Yupatov

UDC 533.6.011.5:621.039.342

Study of cylindrical Couette flow at Knudsen numbers $Kn = 10^{-2}$-1 is not only a classical problem of rarefied gas dynamics, but is also of practical interest [1, 2]. In the case where the outer cylinder is fixed while the inner one rotates with a velocity equivalent to a Mach number $M \le 1$, the flow has been studied experimentally [3] and numerically, both by direct statistical modeling [1], and by solution of model kinetic equations [2, 4]. At supersonic inner cylinder velocities [5, 6] analyzed the effect on flow characteristics of the Mach number and gap size. Significanty fewer studies exist for the case where the outer cylinder rotates while the inner is at rest. A numerical solution of the Boltzmann equation was found for that problem in [2] for the BGK model. Flow of a rarefied binary gas mixture in a planar gap was studied for various ratios of component masses and concentrations in [7], which obtained velocity distributions and components of the viscous stress tensor in the gap.

1. The present study will perform a numerical investigation of the flow of a rarefied binary gas mixture with molecular masses $\mu_1 = 300$ and $\mu_2 = 400$ in the gap between coaxial cylinders using the direct statistical modeling method of [8]. The outer cylinder of radius r_2 rotates with an angular velocity ω_2 ($M = \omega_2 r_2/(\gamma RT_0/\mu_2)^{1/2} = 3$), and the indices 1 and 2 below will refer to quantities defined on the surfaces of the inner and outer cylinders, respectively; T_0 is the temperature of both cylinders; R is the ideal gas constant; γ is the adiabatic index for the heavy gas, equal to unity; $Kn = \lambda_2/(r_2 - r_1)$ was varied over the

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 139-141, September-October, 1990. Original article submitted May 15, 1989.